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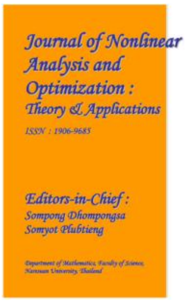

A MATHEMATICAL MODEL OF AN INVENTORY SYSTEM FOR ITEM DETERIORATION WITH TWO OR MORE WAREHOUSE AND SHORTAGES.

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***Abstract* :**
A production-inventory model for a deteriorating item with time-varying demand and fully backlogged shortages is developed for a two warehouse system. For display and storage of inventory, management hires one warehouse of finite capacity at the market place, called own warehouse abbreviated as OW and another warehouse with large capacity as it may be required at a distance place from the market, called rented warehouse abbreviated as RW. Though the time of transportingitems from RW to OW is ignored the transportation cost for transporting items is takento be dependent on the transported amount. Here the objective is to minimize the totalcost for a finite planning horizon. A genetic algorithm (GA) is designed to determinethe optimum number of production cycles and the cycle times within a finite planninghorizon. In this GA a subset of better children is included with the parent populationfor next generation and size of this subset is a percentage of the size of its parent set.Performance of this GA with respect to some other GAs is compared. Two particular cases (i) with non-deteriorating items and (ii) without shortages are also investigated.Finally, to illustrate the model and to show the effectiveness of the proposed approach,a numerical example is provided. With respect to the demand parameters, a sensitivity analysis is performed and presented. In this paper, we have pointed out that the expression of Lee and Hsu (2009) can be obtained as a particularcase.

Keywords
Inventory · Two-warehouse · Time-varying demand · Shortages Deterioration · Genetic algorithm.

1 Introduction :-

Demand is defined as the number of units of an item required by the customers ina unit time. Generally, it usually depends on decisions of people outside the organization. It may be constant or vary with stock, initial lot size, price or time. Normally, demand of seasonal products such as rice, vegetable, etc. increases with time. Several researchers like Silver and Meal (1969), Hariga (1994), Goyal et al. (1996), Kar et al. (2001), Ghosh and Chakrabarty (2009), Mishra and Singh (2011) etc. have analysed inventory models considering time-varying demand.

Natural deterioration of products in the absence of proper preservation conditionsis an important feature of real-life inventory system. In general, deterioration is defined as decay, damage, spoilage etc. that results in decrease of usefulness from theoriginal one. Food grains, vegetables, fruits, etc. are examples of such products. Several authors such as Mandal and Maiti (1999), Kar et al. (2006) and others developed inventory models with different types of deterioration i.e. constant, random or imprecise. Goyal and Giri (2001) presented a survey of research papers listing all the papers with deterioration.

The time period over which the inventory level will be controlled is called the timehorizon. This parameter may be finite or infinite depending upon the nature of the inventory system for the commodity. Normally, inventory models are developed withthe assumption that life time of the product is infinite. According to Gurnani (1985) and Chung and Kim (1989), the assumption of an infinite planning horizon is not realistic due to several reasons such as variation of inventory costs, changes in product specifications and designs, technological changes, etc. Moreover, for seasonal products like fruits, vegetables, warm garments, etc., the business period is rather finite, not infinite. Bhunia and Maiti (1997), Mahapatra and Maiti (2006) and others developed some EOQ models in which time horizon has been considered as finite.

Most of the traditional inventory models are developed by considering a single warehouse (own warehouse) with unlimited capacity. But, in real situation (e.g. in thebusy market place like super market, corporation market, etc.) the capacity of a warehouse is limited. Therefore, the existing single warehouse inventory models are unsuitable for the situation that needs to store a large stock. In fact, there exists manypractical situations that force inventory managers to hold more units than he/she can store in OW. For example, when an attractive price discount for bulk purchase is available or the cost of procuring goods may be higher than the other related costs orthe demand of the item may be very high etc., management decides to purchase (or produce) a large amount of units at a time. Then for storing the excess units, one (or more) additional warehouse(s) is hired on rental basis. This warehouse (RW) may belocated near the OW or a little away from it. Further, the inventory costs (carrying cost and deterioration cost) in RW is usually higher than those in OW due to additional cost of maintenance, material handling, better preserving facilities etc. Moreover, units should be always available at OW for the convenience of business as the actualservice to the customer is carried out at OW only. Hence, the units are first stored in OW and once OW is fulfilled, the units are kept in RW. But RW is cleared first as OWis kept full transferring the units continuously from RW to OW.

In 1976 Hartley first introduced the two warehouse problem in his book 'Operations Research-A Managerial Emphasis'. In his analysis, he ignored the cost of transportation for transferring the goods from RW to OW. Sarma (1987) extended Hartley's model to cover the transportation cost from RW to OW that is considered to be a fixed constant independent of the quantity being transported. Goswami and Chaudhuri (1992) further developed the model with or without shortages by assumingthat demand varies over time with linearly increasing trend and the transportation costfrom RW to OW depends on the quantity being transported. Next Pakkala and Achary(1992a, 1992b) extended the two warehouse models with finite rate of replenishmentand shortages taking time as discrete and continuous variable respectively. In their models, the scheduling period was taken as constant but the transportation cost for transferring the stocks from RW to OW was not taken into account. Besides, the ideasof time varying demand for deteriorating units with two storage facilities were considered by Benkherouf (1997) and Bhunia and Maiti (1997, 1998). All the above models were developed under the assumption that inventory is to be released directlyand continuously in each warehouse. Murdeshwar and Sathi (1985), Pakkala and Achary (1994) considered bulk release model that inventory in RW first transformedto OW before it release to the customer. However, the above models are based on infinite planning horizon with demand parameters reset at the beginning of each cycle. Kar et al. (2001) developed a two-storage inventory model with linear time dependentdemand over a finite time horizon. Lee and Ma (2000) proposed a two-warehouse model and a heuristic solution of equal production cycle times with a general time- dependent demand function and a finite planning horizon. Recently, Lee and Hsu (2009) extended the Lee and Ma's model by considering finite rate of replenishmentand use variable production cycle time (VPCT) approach to determine the number of production cycles and the successive production cycle times within a finite planning horizon.

In this paper, a deterministic two-warehouse inventory model for a deteriorating item is developed with finite replenishment over a finite time horizon. Here we extend the model of Lee and Hsu (2009), to incorporate complete backlogging. The model has been defined as a cost minimization model to determine the optimum number of production cycles and successive production cycle times over a finite planning horizon. A genetic algorithm with varying population size approach is used to solved the model. Moreover, the performance of the proposed algorithm is compared with conventional GA in numerical illustrations. The highlights of the Paper • In this paper,a deterministic two-warehouse (RW,OW) inventory model for a deteriorating item isdeveloped.

- For display and storage of inventory, management hires OW of finite capacity at the market place and RW with large capacity as it may be required at a distance place from the market.
- Time-varying demand and fully backlogged shortages is developed for two warehouse system.

- Time of transporting items from RW to OW is ignored.
- The transportation cost for transporting items is taken to be dependent on the transported amount.
- The model has been defined as a cost minimization model to determine the optimum number of production cycles and successive production cycle times overa finite planning horizon.
- A genetic algorithm with varying population size approach is used to solved the model.

2 Assumptions and notations

The mathematical model in this paper is developed on the basis of followingassumptions and notations:

Assumptions

(i) Inventory system involves two warehouses OW and RW and only one itemandis a self production system.
(ii) The time horizon is finite.
(iii) The time horizon accommodates N full cycles.
(iv) Shortages are allowed.
(v) Lead time is zero.
(vi) The OW has a fixed capacity whereas the RW has unlimited capacity.(vii) Set-up time is negligible.
(viii) Production rate is known and constant.
(ix) The constant fraction of on hand inventory gets deteriorated per unit time.(x)Transportation cost be negligible.
(xi) The inventory carrying cost in RW is higher than that in OW.

Notations

(i) P = Production rate in each cycle.
(ii) W = Capacity of OW.
(iii) W1 = Maximum shortages allowed.
(iv) qi1(t) = On-hand inventory of the item at time t for i^{th} cycle, whenshortages are allowed.
(v) qi2(t) = On-hand inventory of the item at time t for i^{th} cycle, in OW.
(vi) qi3(t) = On-hand inventory of the item at time t for i^{th} cycle, in RW.
(vii) C1 = The set-up cost per production run.
(viii) C2 = Cost of a deteriorated unit.
(ix) Cs = The shortage cost per unit per unit time.
(x) Cow = The carrying cost per inventory unit held in OW per unit oftime.
(xi) Crw = The carrying cost per inventory unit held in RW per unit oftime.

From assumption (xi) we have Crw > Cow.

(xii) f(t) = The demand rate at time t, f(t) < P
$(xiii)$ θ_1, θ_2 = constant deterioration rate in OW and RW respectively, where$0<\theta_1 < 1, 0 < \theta_2 < 1$
(xiv) H = Total planning horizon.
(xv) N = The number of production cycle during the entire time horizon H.
(xvi) m = Boundary cycle number when switching from L2 to L1 in the case oftime increasing demand.
(xvii) SL2 = {i : both OW and RW are used in cycle i,i = 1,2,...,,m}.
(xviii) SL1 = {j : only OW is used in cycle j,j = m + 1,m + 2,...,N}.
(xix) ti0 = The beginning time of the i^{th} production cycle, i ∈ SL2.
(xx) ti1 = The initial time of the i^{th} cycle in OW, for all i ∈ SL2.
(xxi) ti2 = The end of the i^{th} cycle to build up W units in OW for all i, i ∈ SL2.
(xxii) ti3 = The end of production at the i^{th} cycle for all i, i ∈ SL2.
(xxiii) ti4 = The end of depletion of all inventory units in RW at the i^{th} cycle for alli, i ∈ SL2.
(xxiv) ti5 = The end of depletion of all inventory units in OW at the i^{th} cycle for alli, i ∈ SL2.
(xxv) Ti = The end of time of the i^{th} production cycle, i ∈ SL2.
(xxvi) qj1(t) = On-hand inventory of the item at time t for j^{th} cycle, when shortagesare allowed.

(xxvii) $q_{j2}(t)$ = On-hand inventory of the item at time t for j^{th} cycle, in OW.

(xxviii) t_{j0} = The beginning time of the j^{th} production cycle, $j \in S_{L1}$.

(xxix) t_{j1} = The initial time of the j^{th} cycle in OW, for all $j \in S_{L1}$.

(xxx) t_{j2} = The end of production at the j^{th} cycle for all i, $j \in S_{L1}$.

(xxxi) t_{j3} =The end of depletion of all inventory units in OW at the j^{th} cycle for allj ,$j \in S_{L1}$.

(xxxii) T_j = The end of time of the j^{th} production cycle, $j \in S_{L1}$. (xxxiii) TC = Total inventory cost during H.

3 Model formulation

In the development of the two warehouse production model, we assume that there areN cycles during the time horizon H. At the beginning of the i^{th} cycle, W_1 units of backorders are carried over from the previous cycle. The production run begins at $t = t_{i0}$ and while production and demand occur simultaneously, backorders are made up to $t = t_{i1}$. Inventory items in OW begin to accumulate up to W units with deterioration. After $t = t_{i2}$ the produced quantity exceeding W must be stored in RW and productioncontinuous up to $t = t_{i3}$ (cf. Fig. 1). At the end of production, $t = t_{i3}$ the inventory in RW would be depleted due to demand and deterioration and it vanishes at. During $t = t_{i1}$ and $t = t_{i4}$, inventory in OW are also lowered at a level below due to deterioration only. The remaining stock in OW are then fully depleted at $t = t_{i5}$ due to both demand and deterioration and shortages starts at $t = t_{i5}$ and continuous upto time $t = T_i$ when next cycle begins. This cycle repeats again and again. Similarly, the j^{th} cycles occur only within OW.

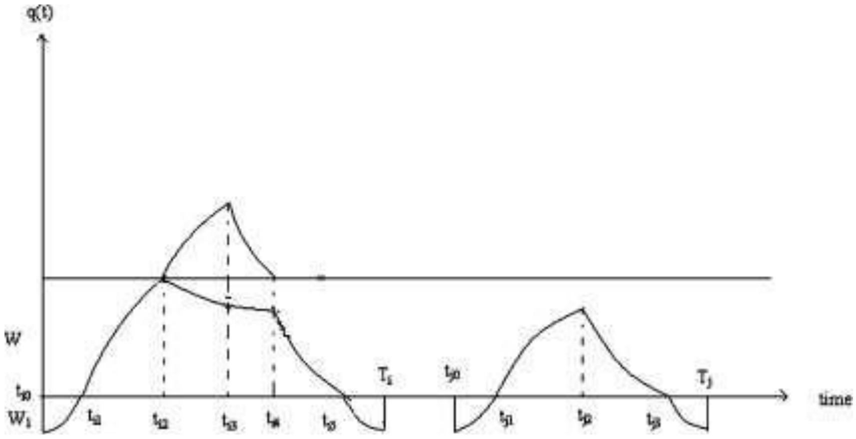

Fig. 1 Graphical representation of a two-warehouse production system with time varying demand

For those cycles of L2, the differential equations describing the inventory level aregiven as follows:

$$\frac{dq_{i1}(t)}{dt} = \begin{cases} P - f(t), & t_{i0} \le t \le t_{i1} \\ -f(t), & t_{i5} \le t \le T_i, \end{cases} \quad (1)$$

and

$$\frac{dq_{i2}(t)}{dt} = \begin{cases} P - f(t) - \theta_1 q_{i2}(t), & t_{i1} \le t \le t_{i2} \\ -\theta_1 q_{i2}(t), & t_{i2} \le t \le t_{i4} \\ -f(t) - \theta_1 q_{i2}(t), & t_{i4} \le t \le t_{i5} \end{cases} \quad (2)$$

$$\frac{dq_{i3}(t)}{dt} = \begin{cases} P - f(t) - \theta_2 q_{i3}(t), & t_{i2} \le t \le t_{i3} \\ f(t) \quad \theta_2 q_{i3}(t), & t_{i3} \le t \le t_{i4}. \end{cases} \quad (3)$$

Using boundary conditions that $q_{i1}(t_{i0}) = -W_1$, $q_{i1}(t_{i1}) = 0$, $q_{i1}(t_{i5}) = 0$, $q_{i2}(t_{i1}) = 0$, $q_{i2}(t_{i2}) = W$, $q_{i2}(t_{i5}) = 0$, $q_{i3}(t_{i2}) = 0$, and $q_{i3}(t_{i4}) = 0$, the above equations can be solved for $f(t) = a.e^{bt}$.

The solutions of the differential equations (1) are given by,

$$q_{i1}(t) = [\frac{a}{b}(e^{bt_{i1}} - e^{bt}) + P(t - t_{i1}), t_{i0} \leq t \leq t_{i1}$$

$$[\frac{a}{b}(e^{bt} - e^{bt_{i5}}, t_{i5} \leq t \leq T_i \tag{4}$$

the solutions of the differential equations (2) are given by,

$$qi2(t) = \frac{P}{\theta_1}[1 - e^{\theta_1(t_{i1}-1)}] - \frac{a}{b+\theta_1}[e^{bt} - e^{\theta_1(t_{i1}-t)+bt_{i1}}]$$
$$[W. e^{\theta_1(t_{i2}-t)}, \frac{a}{b+\theta_1}[e^{(b+\theta_1)t_{i5}-\theta_1 t} - e^{bt}], \tag{5}$$

and the solutions of the differential equations (3) are given by,

$$q_{i3}(t) = \begin{cases} \frac{P}{\theta_2}\{1 - e^{\theta_2(t_{i2}-t)}\} - \frac{a}{b+\theta_2}\{e^{bt} - e^{\theta_2(t_{i2}-t)+bt_{i2}}\}, & t_{i2} \leq t \leq t_{i3} \\ \frac{a}{b+\theta_2}\{e^{(b+\theta_2)t_{i4}-\theta_2 t} - e^{bt}\}, & t_{i3} \leq t \leq t_{i4}. \end{cases} \tag{6}$$

Using the initial condition, at $t = t_{i0}$, $q_{i1}(t) = -W_1$ from (4) we get,

$$-W_1 = P(t_{i0} - t_{i1}) - \frac{a}{b}(e^{bt_{i0}} - e^{bt_{i1}})$$
$$\Rightarrow P(t_{i0} - t_{i1}) - \frac{a}{b}(e^{bt_{i0}} - e^{bt_{i1}}) + W_1 = 0$$

(7)

Using the continuity condition, at $t = t_{i2}$, from (5) we get,

$$\frac{P}{\theta_1}[1 - e^{\theta_1(t_{i1}-t_{i2})}] - \frac{a}{\quad}[e^{bt_{i2}} \quad \tag{8}$$

Using the continuity condition, at $t = t_{i3}$, from (6) we get,

$$P\ \frac{P}{\theta_1}[1 - e^{\theta_2(t_{i2}-t_{i3})}] - \frac{a}{b+\theta_2}[e^{bt_{j3}} - e^{\theta_2(t_{i2}-t_{i3})+bt_{i2}}]$$
$$- + \frac{a}{b+\theta_2}[e^{(b+\theta_2)t_{i4}-\theta_2 t_{i3}} - e^{bt_{i3}}]. \tag{9}$$

Using the continuity condition, at $t = t_{i4}$, from (5) we get,

$$W. e^{\theta_1(t_{i2}-t_{i4})} = \frac{a}{b-\theta_1}[e^{(b+\theta_1)t_{i5}-\theta_1 t_{i4}} - e^{bt_{i4}}] \tag{10}$$

Using the boundary condition, at $t = T_i$, $q_{i1}(t) = -W_1$ from (4) we get,

$$-W_1 = \frac{a}{b}\{e^{bt_{i5}} - e^{bT_i}\}. \tag{11}$$

Now the inventory items held in RW can be derived as

$$
\begin{aligned}
q_{RW,i} &= \int_{t_{i2}}^{t_{i3}} q_{i3}(t)\,dt + \int_{t_{i3}}^{t_{i4}} q_{i3}(t)\,dt \\
&= \int_{t_{i2}}^{t_{i3}} \left[\frac{P}{\theta_2}\left\{1 - e^{\theta_2(t_{i2}-t)}\right\} - \frac{a}{b+\theta_2}\left\{e^{bt} - e^{\theta_2(t_{i2}-t)+bt_{i2}}\right\}\right] dt \\
&\quad + \int_{t_{i3}}^{t_{i4}} \frac{a}{b+\theta_2}\left[e^{(b+\theta_2)t_{i4}-\theta_2 t} - e^{bt}\right] dt \\
&= \frac{P}{\theta_2}\left[(t_{i3}-t_{i2}) + \frac{1}{\theta_2}\left\{e^{\theta_2(t_{i2}-t_{i3})} - 1\right\}\right] - \frac{a}{b+\theta_2}\left[\frac{1}{b}\left(e^{bt_{i3}} - e^{bt_{i2}}\right)\right. \\
&\quad \left. + \frac{1}{\theta_2}\left\{e^{(b+\theta_2)t_{i2}-\theta_2 t_{i3}} - e^{bt_{i2}}\right\}\right] + \frac{a}{b+\theta_2}\left[\frac{1}{\theta_2}\left\{e^{(b+\theta_2)t_{i4}-\theta_2 t_{i3}}\right.\right. \\
&\quad \left.\left. - e^{bt_{i4}}\right\} + \frac{1}{b}\left(e^{bt_{i3}} - e^{bt_{i4}}\right)\right].
\end{aligned}
$$

Similarly, the inventory levels held in OW can be derived as

$$
\begin{aligned}
q_{OW,i} &= \int_{t_{i1}}^{t_{i2}} q_{i2}(t)\,dt + \int_{t_{i2}}^{t_{i4}} q_{i2}(t)\,dt + \int_{t_{i4}}^{t_{i5}} q_{i2}(t)\,dt \\
&= \int_{t_{i1}}^{t_{i2}} \left[\frac{P}{\theta_1}\left\{1 - e^{\theta_1(t_{i1}-t)}\right\} - \frac{a}{b+\theta_1}\left\{e^{bt} - e^{(\theta_1+b)t_{i1}-\theta_1 t}\right\}\right] dt \\
&\quad + \int_{t_{i2}}^{t_{i4}} W e^{\theta_1(t_{i2}-t)}\,dt + \int_{t_{i4}}^{t_{i5}} \frac{a}{b+\theta_1}\left[e^{(b+\theta_1)t_{i5}-\theta_1 t} - e^{bt}\right] dt \\
&= \frac{P}{\theta_1}\left[(t_{i2}-t_{i1}) + \frac{1}{\theta_1}\left\{e^{\theta_1(t_{i1}-t_{i2})} - 1\right\}\right] - \frac{a}{b+\theta_1}\left[\frac{1}{b}\left(e^{bt_{i2}} - e^{bt_{i1}}\right)\right. \\
&\quad \left. + \frac{1}{\theta_1}\left\{e^{(b+\theta_1)t_{i1}-\theta_1 t_{i2}} - e^{bt_{i1}}\right\}\right] - \frac{W}{\theta_1}\left(e^{\theta_1(t_{i2}-t_{i4})} - 1\right) \\
&\quad + \frac{a}{b+\theta_1}\left[\frac{1}{\theta_1}\left\{e^{(b+\theta_1)t_{i5}-\theta_1 t_{i4}} - e^{bt_{i5}}\right\} + \frac{1}{b}\left(e^{bt_{i4}} - e^{bt_{i5}}\right)\right]. \qquad (13)
\end{aligned}
$$

The inventory items deteriorated in cycle i is given by

$$
\begin{aligned}
D_i &= \int_{t_{i1}}^{t_{i2}} \theta_1 q_{i2}(t)\,dt + \int_{t_{i2}}^{t_{i4}} \theta_1 q_{i2}(t)\,dt + \int_{t_{i4}}^{t_{i5}} \theta_1 q_{i2} \\
&\quad + \int_{t_{i2}}^{t_{i3}} \theta_2 q_{i3}(t)\,dt + \int_{t_{i3}}^{t_{i4}} \theta_2 q_{i3}(t)\,dt \\
&= \theta_1 . q_{OW,i} + \theta_2 . q_{RW,i} . \qquad 14
\end{aligned}
$$

The inventory item shortages in cycle i is given by

$$
\begin{aligned}
S_i &= \int_{t_{i0}}^{t_{i1}} q_{i1}(t)\,dt + \int_{t_{i5}}^{T_i} q_{i1}(t)\,dt \\
&= \int_{t_{i0}}^{t_{i1}} \left\{\frac{a}{b}\left(e^{bt_{i1}} - e^{bt}\right) + P(t - t_{i1})\right\} dt + \int_{t_{i5}}^{T_i} \frac{a}{b}\left(e^{bt} - e^{bt_{i5}}\right) dt \\
&= \frac{a}{b}\left\{e^{bt^{i1}}(t_{i1} - t_{i0}) - \frac{1}{b}\left(e^{bt_{i1}} - e^{bt_{i0}}\right)\right\} + P\left\{\frac{1}{2}\left(t_{i1}^2 - t_{i0}^2\right) - t_{i1}(t_{i1} - t_{i0})\right\} \\
&\quad + \frac{a}{b}\left\{\frac{1}{b}\left(e^{bT_i} - e^{bt_{i5}}\right) - e^{bt_{i5}}(T_i - t_{i5})\right\}. \qquad (15)
\end{aligned}
$$

Again for those cycles using OW only (L1), the differential equations describing theinventory level are given as follows:

$$
\frac{dq_{j1}(t)}{dt} = \begin{cases} P - f(t), & t_{j0} \le t \le t_{j1} \\ -f(t), & t_{j3} \le t \le T_j \end{cases}
$$

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and

$$
\frac{dq_{j2}(t)}{dt} = \begin{cases} P - f(t) - \theta_1 q_{j2}(t), & t_{j1} \le t \le t_{j2} \\ -\theta_1 q_{j2}(t) - f(t), & t_{j2} \le t \le t_{j3} \end{cases}
$$

(17)

Using boundary conditions that qj1(tj1) = 0, qj1(tj3) = 0, qj2(tj1) = 0 and qj2(tj3) = 0, theabove

equations can be solved for above case as follows:
The solutions of the differential equations (16) are given by,

$$q_{j1}(t)=\begin{cases}\frac{a}{b}(e^{bt_{j1}}-e^{bt})+P(t-t_{j1}), & t_{j0}\le t\le t_{j1}\\ -\frac{a}{b}(e^{bt}-e^{bt_{j3}}), & t_{j3}\le t\le T_j\end{cases} \tag{18}$$

the solutions of the differential equations (17) are given by,

$$q_{j2}(t)=\begin{cases}\frac{P}{\theta_1}\{1-e^{\theta_1(t_{j1}-t)}\}-\frac{a}{b+\theta_1}\{e^{bt}-e^{\theta_1(t_{j1}-t)+bt_{j1}}\}, & t_{j1}\le t\le t_{j2}\\ \frac{a}{b+\theta_1}\{e^{(b+\theta_1)t_{j3}-\theta_1 t}-e^{bt}\}, t_{j2}\le t\le t_{j3}.\end{cases} \tag{19}$$

Using the boundary condition, at t = tj0, qj1(t) = −W1 from (18) we get,

$$-W_1=P(t_{j0}-t_{j1})-\frac{a}{b}(e^{bt_{j0}}-e^{bt_{j1}})$$

$$\Rightarrow P(t_{j0}-t_{j1})-\frac{a}{b}(e^{bt_{j0}}-e^{bt_{j1}})+W_1=0 \quad . \tag{20}$$

Using the continuity condition, at t = tj2, from (19) we get,

$$\frac{P}{\theta_1}[1-e^{\theta_1(t_{j1}-t_{j2})}]-\frac{a}{b+\theta_1}[e^{bt_{j2}}-e^{\theta_1(t_{j1}-t_{j2})+bt_{j1}}]$$

$$=\frac{a}{b+\theta_1}[e^{(b+\theta_1)t_{j3}-\theta_1 t_{j2}}-e^{bt_{j2}}]$$

(21)

Using the boundary condition, at t = Tj, qj1(t) = −W1 from (18) we get,

$$-W_1=\frac{a}{b}\{e^{bt_{j3}}-e^{bT_j}\}. \tag{22}$$

Here the inventory items held in RW can be derived as
qRW,j = 0.
Similarly, the inventory levels held in OW can be derived as

$$\begin{aligned}q_{OW,j}&=\int_{t_{j1}}^{t_{j2}}q_{j2}(t)\,dt+\int_{t_{j2}}^{t_{j3}}q_{j2}(t)\,dt\\&=\int_{t_{j1}}^{t_{j2}}\left[\frac{P}{\theta_1}\{1-e^{\theta_1(t_{j1}-t)}\}-\frac{a}{b+\theta_1}\{e^{bt}-e^{(\theta_1+b)t_{j1}-\theta_1 t}\}\right]dt\\&\quad+\int_{t_{j2}}^{t_{j3}}\frac{a}{b+\theta_1}[e^{(b+\theta_1)t_{j3}-\theta_1 t}-e^{bt}]\,dt\\&=\frac{P}{\theta_1}\left[(t_{j2}-t_{j1})+\frac{1}{\theta_1}\{e^{\theta_1(t_{j1}-t_{j2})}-1\}\right]-\frac{a}{b+\theta_1}\left[\frac{1}{b}(e^{bt_{j2}}-e^{bt_{j1}})\right.\\&\quad\left.+\frac{1}{\theta_1}\{e^{(b+\theta_1)t_{j1}-\theta_1 t_{j2}}-e^{bt_{j1}}\}\right]\\&\quad+\frac{a}{b+\theta_1}\left[\frac{1}{\theta_1}\{e^{(b+\theta_1)t_{j3}-\theta_1 t_{j2}}-e^{bt_{j3}}\}+\frac{1}{b}(e^{bt_{j2}}-e^{bt_{j3}})\right]. \qquad (24)\end{aligned}$$

(23)

The inventory items deteriorated in cycle j is given by

$$D_j=\int_{t_{j1}}^{tj2}\theta_1 q_{j2}(t)dt+\int_{t_{j2}}^{t_{j3}}\theta_1 q_{j2}(t)dt$$

$$=\theta_1.\,qow,j$$

(25)

The inventory item shortages in cycle j is given by

$$S_j = \int_{t_{j0}}^{t_{j1}} q_{j1}(t)\,dt + \int_{t_{j3}}^{T_j} q_{j1}(t)\,dt$$
$$= \int_{t_{j0}}^{t_{j1}} \left\{ \frac{a}{b}\left(e^{bt_{j1}} - e^{bt}\right) + P(t - t_{j1}) \right\} dt + \int_{t_{j3}}^{T_j} \frac{a}{b}\left(e^{bt} - e^{bt_{j3}}\right) dt$$
$$= \frac{a}{b}\left\{ e^{bt_{j1}}(t_{j1} - t_{j0}) - \frac{1}{b}\left(e^{bt_{j1}} - e^{bt_{j0}}\right) \right\} + P\left\{ \frac{1}{2}\left(t_{j1}^2 - t_{j0}^2\right) - t_{j1}(t_{j1} - t_{j0}) \right\}$$
$$+ \frac{a}{b}\left\{ \frac{1}{b}\left(e^{bT_j} - e^{bt_{j3}}\right) - e^{bt_{j3}}(T_j - t_{j3}) \right\}. \quad (26)$$

The total system cost during the planning horizon H can then be expressed as

$$TC = NC_1 + C_{rw} \sum_{i \in S_{L_2}} q_{RW,i} + C_{ow} \sum_{i \in S_{L_2}} q_{OW,i} + C_2 \sum_{i \in S_{L_2}} D_i$$
$$+ C_s \sum_{i \in S_{L_2}} S_i + C_{ow} \sum_{j \in S_{L_1}} q_{OW,j} + C_2 \sum_{j \in S_{L_1}} D_j + C_s \sum_{j \in S_{L_1}} S_j. \quad (27)$$

So, the above model is formulated as the unconstrained minimization problem of totalcost i.e.

Minimize TC.

Particular cases

Now, two particular cases of the general model are further investigated:

(i) With non-deteriorating items, shortages allowed but fully backlogged.

When there are non-deteriorating items in both warehouses, i.e., when both θ_1 and θ_2 approach zero, the following results are derived:

Under those cycles of L2, we have when $\theta_2 \to 0$, then

$$q_{RW,i} = P\left\{ \frac{1}{2}\left(t_{i3}^2 - t_{i2}^2\right) - t_{i2}(t_{i3} - t_{i2}) \right\} - \frac{a}{b}\left\{ \frac{1}{b}\left(e^{bt_{i3}} - e^{bt_{i2}}\right) - (t_{i3} - t_{i2})e^{bt_{i2}} \right\}$$
$$+ \frac{a}{b}\left\{ (t_{i4} - t_{i3})e^{bt_{i4}} + \frac{1}{b}\left(e^{bt_{i3}} - e^{bt_{i4}}\right) \right\}. \quad (28)$$

When $\theta_1 \to 0$, then

$$q_{OW,i} = P\left\{ \frac{1}{2}\left(t_{i2}^2 - t_{i1}^2\right) - t_{i1}(t_{i2} - t_{i1}) \right\} - \frac{a}{b}\left\{ \frac{1}{b}\left(e^{bt_{i2}} - e^{bt_{i1}}\right) - (t_{i2} - t_{i1})e^{bt_{i1}} \right\}$$
$$+ W(t_{i4} - t_{i2}) + \frac{a}{b}\left\{ (t_{i5} - t_{i4})e^{bt_{i5}} + \frac{1}{b}\left(e^{bt_{i4}} - e^{bt_{i5}}\right) \right\}. \quad (29)$$

When $\theta_1, \theta_2 \to 0$, then

$D_i = 0$.(30)

There is no effect in S_i, since there is no inventory.Under those cycles of L1, we have when $\theta_2 \to 0$, then

$q_{RW,j} = 0.$ (31)

When $\theta_1 \to 0$, then

$$q_{OW,i} = P\left\{ \frac{1}{2}\left(t_{j2}^2 - t_{j1}^2\right) - t_{j1}(t_{j2} - t_{j1}) \right\} - \frac{a}{b}\left\{ \frac{1}{b}\left(e^{bt_{j2}} - e^{bt_{j1}}\right) - (t_{j2} - t_{j1})e^{bt_{j1}} \right\}$$
$$+ \frac{a}{b}\left\{ (t_{j3} - t_{j2})e^{bt_{j3}} + \frac{1}{b}\left(e^{bt_{j2}} - e^{bt_{j3}}\right) \right\}. \quad (32)$$

When $\theta_1, \theta_2 \to 0$, then

Dj = 0. (33)

There is no effect in Sj, since there is no inventory.
When θ1,θ2 → 0, then the total system cost can be written as

$$TC = NC_1 + C_{rw}\sum_{i\in S_{L_2}} q_{RW,i} + C_{ow}\sum_{i\in S_{L_2}} q_{OW,i} + C_s\sum_{i\in S_{L_2}} S_i + C_{ow}\sum_{j\in S_{L_1}} q_{OW,j} + C_s\sum_{j\in S_{L_1}} S_j \quad (34)$$

(i) Without shortages, but with deteriorating items in both OW and RW.The inventory items held in RW can be derived as

$$q_{RW,i} = \frac{P}{\theta_2}\left[(t_{i2}-t_{i1}) + \frac{1}{\theta_2}\left\{e^{\theta_2(t_{i1}-t_{i2})}-1\right\}\right] - \frac{a}{b+\theta_2}\left[\frac{1}{b}\left(e^{bt_{i2}}-e^{bt_{i1}}\right) + \frac{1}{\theta_2}\left\{e^{(b+\theta_2)t_{i1}-\theta_2 t_{i2}} - e^{bt_{i1}}\right\}\right] + \frac{a}{b+\theta_2}\left[\frac{1}{\theta_2}\left\{e^{(b+\theta_2)t_{i3}-\theta_2 t_{i2}} - e^{bt_{i3}}\right\} + \frac{1}{b}\left(e^{bt_{i2}}-e^{bt_{i3}}\right)\right]. \quad 35$$

Similarly, the inventory levels held in OW can be derived as

$$q_{OW,i} = \frac{P}{\theta_1}\left[(t_{i1}-t_{i0}) + \frac{1}{\theta_1}\left\{e^{\theta_1(t_{i0}-t_{i1})}-1\right\}\right] - \frac{a}{b+\theta_1}\left[\frac{1}{b}\left(e^{bt_{i1}}-e^{bt_{i0}}\right) + \frac{1}{\theta_1}\left\{e^{(b+\theta_1)t_{i0}-\theta_1 t_{i1}} - e^{bt_{i0}}\right\}\right] - \frac{W}{\theta_1}\left(e^{\theta_1(t_{i1}-t_{i3})}-1\right) + \frac{a}{b+\theta_1}\left[\frac{1}{\theta_1}\left\{e^{(b+\theta_1)T_i-\theta_1 t_{i3}} - e^{bT_i}\right\} + \frac{1}{b}\left(e^{bt_{i3}}-e^{bT_i}\right)\right]. \quad (36)$$

The inventory items deteriorated in cycle i is given by
Di = θ1.qOW,i + θ2.qRW,i. 37
The inventory item shortages in cycle i is given by
Si = 0. 38
The inventory items held in RW can be derivedas
qRW,j = 0. 39

Similarly, the inventory levels held in OW can be derived as

$$q_{OW,j} = \frac{P}{\theta_1}\left[(t_{j1}-t_{j0}) + \frac{1}{\theta_1}\left[e^{\theta_1(t_{j0}-t_{j1})}-1\right]\right]$$

$$-\frac{a}{b+\theta_1}\left[\frac{1}{b}\left(e^{bt_{j1}-e^{bt_{j0}}}\right) + \frac{1}{\theta_1}\left(e^{(b+\theta_1)t_{j0}-\theta_1 t_{j1}} - e^{bt_{j0}}\right)\right]$$

$$+\frac{a}{b+\theta_1}\left[\frac{1}{a}\left(e^{(b+\theta_1)T_j-\theta_1 t_{j1}} \quad bT_j \quad 1\right.\right.$$

The inventory items deteriorated in cycle j is given by
Dj = θ1.qOW,j. 4 1
Theinventory item shortages in cycle j is given by
Sj = 0. (42)
The total system cost during the planning horizon H can then be expressed as

$$TC = NC_1 + C_{rw}\sum_{i\in S_{L_2}} q_{RW,i} + C_{ow}\sum_{i\in S_{L_2}} q_{OW,i} + C_2\sum_{i\in S_{L_2}} D_i + C_{ow}\sum_{j\in S_{L_1}} q_{OW,j} + C_2\sum_{j\in S_{L_1}} D_j. \quad (43)$$

It is seen that the expression (43) is the same expression (19) of Lee and Hsu (2009) with $f(t) = a.e^{bt}$.

4 Solution procedure

The above model is solved by using genetic algorithm with varying population size approach, discussed in Sect. 4.1, and also solved by conventional genetic algorithm. Our conventional GA consists of parameters, population size = 50, probability of crossover = 0.5, probability of mutation = 0.2, maximum generation = 50. A real number presentation is used here. In this representation, each chromosome X is a string of n numbers of genes which denote the decision variable. For each chromosome X, every gene, which represents the independent variables, are randomly generated between their boundaries until it is feasible. In this conventional GA, arithmetic crossover and random mutation are applied to generate new offspring's. Our varying population size approach of GA consists of parameters, maximum lifetime of a chromosome = 8, minimum lifetime of a chromosome = 1, probability ofmutation = 0.2, maximum generation = 15, number of solutions = 15.

Genetic algorithm (varying population size approach)

After development of Genetic Algorithm (GA) by Holland (Holland 1975; Michalewicz 1992), it has been extensively used/modified to solve complex decision making problems in different field of science and technology. A GA normally starts with a set of potential solutions (called initial population) of the decision making problem under consideration. Individual solutions are called chromosome. Crossoverand mutation operations happen among the potential solutions to get a new set of solutions and it continues until terminating conditions are encountered. As mentioned earlier behavior and performance of a GA is directly affected by the interaction between the parameters, i.e., selection process of chromosomes for mating pool, pc, pm, etc. Recently, Last and Eyal (2005) developed a GA with varying population size, where chromosomes are classified into young, middle age and old according to theirage and lifetime. Genotype diversity, Phenotype diversity of the final population are obtained to measure the performance of the GA. Following Last and Eyal (2005), here, a GA with varying population size is developed where chromosomes are classified into young, middle age and old (in fuzzy sense) according to their age and lifetime. Following comparison of fuzzy numbers using possibility theory (Dubois and Prade 1980; Liu and Iwamura 1998), here crossover probability is measured as afunction of parent's age interval (a fuzzy rule base on parents age limit is also used for this purpose). In this GA a subset of better children is included with the parent population for next generation and size of this subset is a percentage of the size of itsparent set. Chromosomes with age exceeds lifetime are discarded from the population at the beginning of every iteration. General structure of this GA is presented below.

1. Set iteration counter t = 0 and maximum generation M = M0
2. Randomly generate initial population p(t)
3. Evaluate initial population p(t)
4. While t ≤ M do
5. t = t + 1
6. Increase age of each chromosome
7. For each pair of parents do
8. Determine probability of crossover pc for the selected pair of parent
9. Perform crossover with probability pc
10. For each offspring perform mutation with probability pm.
11. Store offsprings into offspring set

12. End do
13. Select a percent of better offsprings from the offspring set and insert into p(t)
14. Remove from p(t) all individuals with age grater than their lifetime.
15. Evaluate p(t)
16. Remove all offsprings from the offspring set
17. End While
18. End Algorithm

GA procedures for the proposed model

(a) Representation: A 'n dimensional real vector' $X = (x_1,x_2,...,x_n)$ is used torepresent a solution, where $x_1,x_2,...,x_n$ represent n decision variables of the problem.

(b) Initialization: N such solutions $X_i = (x_{i1},x_{i2},...,x_{in})$, i = 1,2,...,N are randomly generated by random number generator within the boundaries of each variable. Value of the objective function due to the solution X_i, is taken as fitness of X_i. Let it be $F(X_i)$. At the time of initialization age of each solution is set to zero. Following Michalewicz (1992) at the time of birth life-time of X_i is computed using the following formula:

$$ifAvgfit \geq F(X_i), lifetime(X_i = Minlt + \frac{K(F(X_i) - Minfit)}{Avgfit - Minfit}$$

$$IfAvgfit < F(X_i), lifetime(X_i) = \frac{Minlt + Maxlt}{2} + \frac{K(F(X_i) - Avgfit,}{Maxfit - Avgfit}$$

where *Maxlt* and *Minlt* are maximum and minimum allowed lifetime of a chromosome, K = (*Maxlt* − *Minlt*)/2. *Maxfit*, *Avgfit*, *Minfit* represent the best, average and worst fitness of the current population. To solve our model it is assumed that *Maxlt* = 8 and *Minlt* = 1, $p_m = 0.2$, $M_0 = 15$, N = 15. According to the age a chromosome can belong to any one of age interval—young, middle-age or old, whose membership function is presented in Fig. 2.

(c) Crossover:

I. Determination of probability of crossover (p_c): Probability of crossover p_c, for a pair of parents (X_i,X_j) is determined as below:

(i) At first age intervals (young, middle-age, old) of X_i and X_j are determined by making possibility measure of fuzzy numbers- young, middle-age, old with respect to their age.

(ii) After determination of age intervals of the parents their crossover probabilityis determined as a linguistic variable (low, medium or high) usinga fuzzy rule base as presented in Table 1. Membership function of these linguistic variables are presented in Fig. 3.

(iii) Center of gravity (Buckley and Eslami 2002) of crossover probability is taken as value of p_c for the parents (X_i,X_j).

Fig.2 Membership function of age intervals

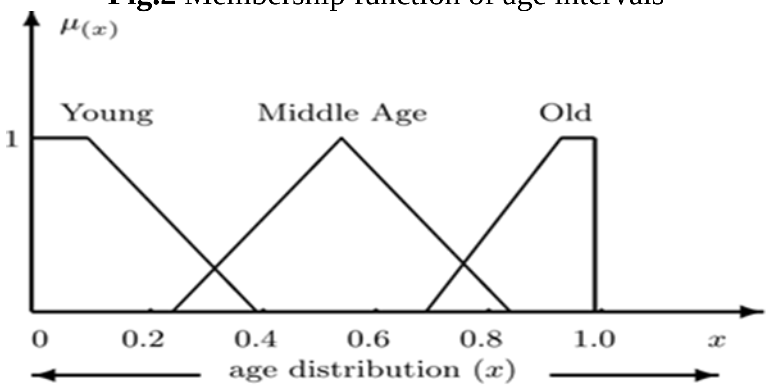

Fig.3 Membership function of crossoverprobabilities

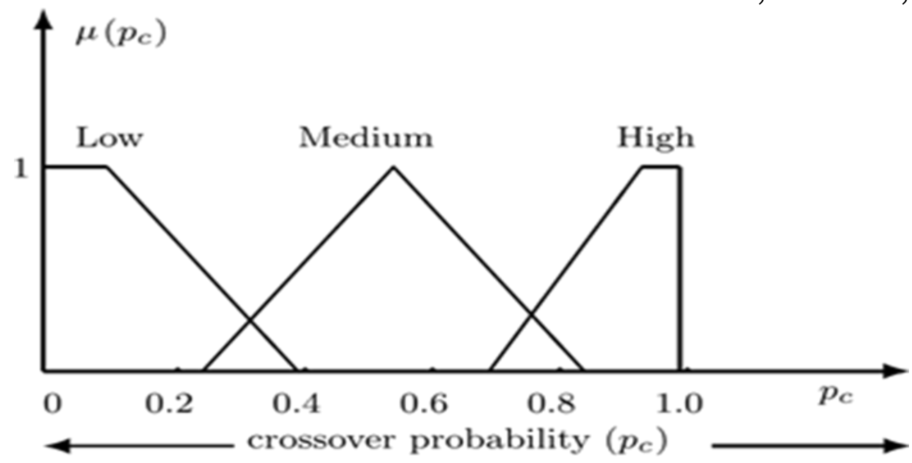

Table 1 Fuzzy rule base for crossover probability

Parent 2	Parent 1		
	Young	Middle-age	Old
Young	Low	Medium	Low
Middle-age	Medium	High	Medium
Old	Low	Medium	Low

II Crossover process: For each pair of parent solutions X_i, X_j a random number c is generated from the range [0,1] and if $c > p_c$ then crossover operation is made on X_i, X_j. To made crossover operation another random number r is generated randomly from the range (0,1) and their off springs Y_1 and Y_2 are obtained by the formula:

$Y_1 = rX_i + (1 - r)X_j$, $\qquad$ $Y_2 = rX_j + (1 - r)X_i$.

(d) Mutation:

(i) **Selection for mutation**: For each offspring generate a random number r from therange [0,1]. If $r < p_m$ then the solution is taken for mutation, where p_m is the probability of mutation.

(ii) **Mutation process**: To mutate a solution $X = (x_1, x_2, ..., x_n)$ select a random integerr in the range [1,n]. Then replace x_r by randomly generated value within the boundary of r^{th} component of X.

(e) Selection of offspring: Maximum population growth in a generation is assumed as fifty percent. So not all off springs are taken into the parent set for next generation. At first offspring set is arranged in descending order in fitness. Then bettersolutions are selected and entered into parent set.

(f) Implementation: With the above function and values the algorithm is implemented using C-programming language.

5 Numerical illustration

An example is presented to illustrate the effect of the inventory model developed herewith the following numerical data: $C_1 = 2500$, $P = 1800$, $C_2 = 50$, $C_s = 0.75$,
$W = 600$, $W_1 = 200$, $\theta_1 = 0.02$, $\theta_2 = 0.01$, $C_{ow} = 5$, $C_{rw} = 8$, $a = 400$, $b = 0$.
H 10.

Using these values, (27), (34) and (43) have been solved using GA with varying population size approach and conventional GA. The results are presented in the following Tables 2, 3, 4, 5.

6.Sensitivity analysis As the time-dependent demand, $f(t) = a.e^{bt}$, the parameters'a' and 'b' influence the whole production-inventory system

Table 2 The optimal solution for two-warehouse model with time increasing demand using genetic algorithm (with varying population size approach [VPSA] and conventional approach [CA])

N	t_{i0}				TC		m
					VPSA	CA	
1	$t_{10} = 0.00$				39271.59	39273.02	1
2	$t_{10} = 0.00$	$t_{20} = 5.12$			34276.56	34278.26	1
3	$t_{10} = 0.00$	$t_{20} = 3.42$	$t_{30} = 6.70$		31616.47	31618.39	1
4	$t_{10} = 0.00$	$t_{20} = 2.94$	$t_{30} = 5.49$	$t_{40} = 8.31$	29914.44	29916.42	1
5	$t_{10} = 0.00$	$t_{20} = 2.81$	$t_{30} = 5.11$	$t_{40} = 6.91$			
	$t_{50} = 8.63$				30062.39	30064.37	1
6	$t_{10} = 0.00$	$t_{20} = 2.77$	$t_{30} = 4.88$	$t_{40} = 6.50$			
	$t_{50} = 7.82$	$t_{60} = 8.10$			32194.63	32196.58	4
7	$t_{10} = 0.00$	$t_{20} = 2.70$	$t_{30} = 4.79$	$t_{40} = 6.43$			
	$t_{50} = 7.70$	$t_{60} = 7.82$	$t_{70} = 9.02$		34002.12	34003.89	4

Table 3 The optimal solution for two-warehouse model with time increasing demand without deterioration using genetic algorithm (with varying population size approach [VPSA] and conventional approach [CA])

N	t_{i0}				TC		m
					VPSA	CA	
1	$t_{10} = 0.00$				39174.59	39176.32	1
2	$t_{10} = 0.00$	$t_{20} = 5.16$			34183.63	34184.89	1
3	$t_{10} = 0.00$	$t_{20} = 3.43$	$t_{30} = 6.74$		31527.84	31528.55	1
4	$t_{10} = 0.00$	$t_{20} = 2.96$	$t_{30} = 6.01$	$t_{40} = 8.11$	29825.62	29827.21	1
5	$t_{10} = 0.00$	$t_{20} = 2.82$	$t_{30} = 5.12$	$t_{40} = 6.92$			
	$t_{50} = 8.63$				29865.63	29867.54	1
6	$t_{10} = 0.00$	$t_{20} = 2.78$	$t_{30} = 4.89$	$t_{40} = 6.54$			
	$t_{50} = 7.83$	$t_{60} = 8.12$			32094.85	32096.45	3
7	$t_{10} = 0.00$	$t_{20} = 2.71$	$t_{30} = 4.79$	$t_{40} = 6.44$			
	$t_{50} = 7.71$	$t_{60} = 8.23$	$t_{70} = 9.03$		33012.76	33014.62	3

values of minimum total costs are presented in Tables 6 and 7 against different valuesof ‘a’ and ‘b’ for N = 1 and N = 2 respectively. In both cases, it is observed from Tables6 and 7 that with the increase of either ‘a’ or ‘b’, demand increases and as result, total cost of the inventory system decreases gradually. This is because, with the increase of demand, deteriorated amount decreases and hence total cost decreases.

6 Discussion

In this paper, we developed a production-inventory model for a deteriorating item with time-varying demand by considering fully backlogged shortages under two warehouse facilities. The proposed model is solved numerically using both GA with

Table 4 The optimal solution for two-warehouse model with time increasing demand without shortages
using genetic algorithm (with varying population size approach [VPSA] and conventional approach [CA])

N	ti0				VPSA	CA	
1	$t_{10} = 0.00$				39194.35	39196.32	1
2	$t_{10} = 0.00$	$t_{20} = 5.15$			34207.66	34209.59	1
3	$t_{10} = 0.00$	$t_{20} = 3.43$	$t_{30} = 6.74$		31546.88	31548.68	1
4	$t_{10} = 0.00$	$t_{20} = 2.96$	$t_{30} = 6.01$	$t_{40} = 8.10$	29845.92	29847.48	1
5	$t_{10} = 0.00$	$t_{20} = 2.82$	$t_{30} = 5.11$	$t_{40} = 6.91$			
	$t_{50} = 8.63$				29883.69	29885.82	1
6	$t_{10} = 0.00$	$t_{20} = 2.78$	$t_{30} = 4.89$	$t_{40} = 6.54$			
	$t_{50} = 7.83$	$t_{60} = 8.12$			32115.49	32117.39	3
7	$t_{10} = 0.00$	$t_{20} = 2.71$	$t_{30} = 4.78$	$t_{40} = 6.43$			
	$t_{50} = 7.71$	$t_{60} = 8.22$	$t_{70} = 9.01$		33032.68	33034.77	3

Table 5Highest stock level for two-warehouse model with time increasing demandusing genetic algorithm (with varying population size approach)

N	Highest stock level (including OW and RW)
1	974.68
2	887.39
3	853.66
4	839.99
5	821.39
6	818.77
7	816.02

Table 6 Sensitivity analysis for two-warehouse model with time increasing demand using genetic algorithm (with varying population size approach) when N = 1

a	b	TC	a	b	TC
500	0.005	39237.20	500	0.01	39212.08
450	0.005	39246.28	450	0.01	39234.62
400	0.005	39283.11	400	0.01	39271.59
350	0.005	39299.54	350	0.01	39292.36
300	0.005	39318.95	300	0.01	39311.44
500	0.015	39178.26	500	0.02	39152.55
450	0.015	39198.30	450	0.02	39171.35
400	0.015	39220.92	400	0.02	39193.44
350	0.015	39240.01	350	0.02	39219.53
300	0.015	39258.66	300	0.02	39237.20

varying population size approach and conventional GA. Two particular cases (i) withnon-deteriorating items ($\theta_1 = 0, \theta_2 = 0$) and (ii) without shortage are also presented andsolved using above two techniques. From the numerical illustration, we have seen that GA with varying population size approach gives the better result (less cost) than the conventional GA for all the cases.

Table 7 Sensitivity analysis for two-warehouse model with time increasing demand using - varying population size approach) when N = 2

a	*b*	*TC*	*a*	*b*
500	0.005	34269.64		0.0 34248.5 5001 6
450	0.005	34277.44		4500.0 34261.5 1 2
400	0.005	34293.73		4000.0 34276.5 1 6
350	0.005	34312.33		3500.0 34291.4 1 7
300	0.005	34321.53		3000.0 34304.7 1 7
500	0.015	34229.64		5000.0 34212.0 2 1
450	0.015	34247.15		4500.0 34230.5 2 5
400	0.015	34264.73		4000.0 34247.9 2 9
350	0.015	34279.52		3500.0 34264.7 2 3
300	0.015	34292.45		3000.0 34282.5 2 7

7 Conclusion

This study presents a two-warehouse (OW and RW) production inventory model fordeteriorating items with time-varying demand and constant production rate. Moreover, it is assumed that the shortages are allowed and backlogged completely. Under these conditions, the problem is formulated as a non-linear programming problem and a genetic algorithm with varying population size is proposed to solve it. Results in this study provide a valuable reference for decision makers in planning andcontrolling the inventory management. Finally, a future study will incorporate more realistic assumptions in the proposed model, such as variable deterioration rate, stochastic nature of demand and production rate.

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